Candidate surname

Other names

Pearson Edexcel International GCSE

Centre Number

Candidate Number

Tuesday 7 January 2020

Morning (Time: 2 hours)

Paper Reference **4MA1/1HR**

Mathematics A
Paper 1HR
Higher Tier

You must have:
Ruler graduated in centimetres and millimetres, protractor, compasses, pen, HB pencil, eraser, calculator. Tracing paper may be used.

Total Marks

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Without sufficient working, correct answers may be awarded no marks.
- Answer the questions in the spaces provided
  – *there may be more space than you need*.
- **Calculators may be used.**
- You must **NOT** write anything on the formulae page.
  Anything you write on the formulae page will gain NO credit.

Information

- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question*.

Advice

- Read each question carefully before you start to answer it.
- Check your answers if you have time at the end.

Turn over ▶

Pearson

International GCSE Mathematics
Formulae sheet – Higher Tier
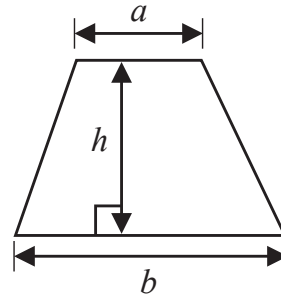
Arithmetic series

Sum to n terms, $S_n = \dfrac{n}{2}\left[2a + (n-1)d\right]$
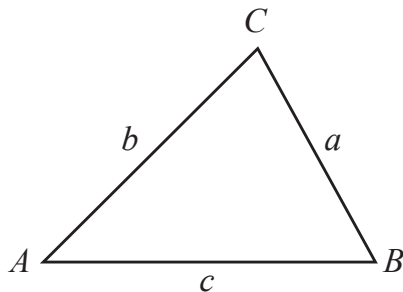
The quadratic equation

The solutions of $ax^2 + bx + c = 0$ where $a \neq 0$ are given by:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Area of trapezium $= \dfrac{1}{2}(a + b)h$
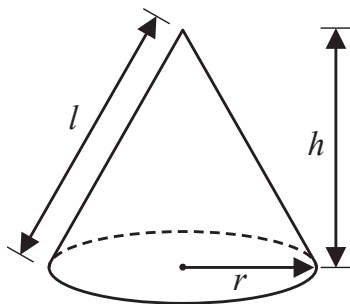

Trigonometry

In any triangle ABC

Sine Rule $\quad \dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

Cosine Rule $\quad a^2 = b^2 + c^2 - 2bc\cos A$

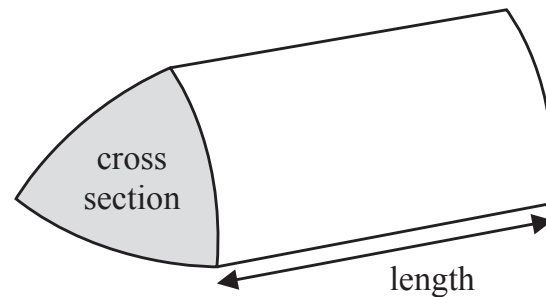**Area of triangle** $= \dfrac{1}{2}ab\sin C$

Volume of cone $= \dfrac{1}{3}\pi r^2 h$
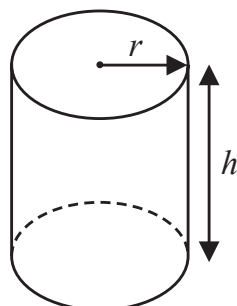
Curved surface area of cone $= \pi r l$

Volume of prism
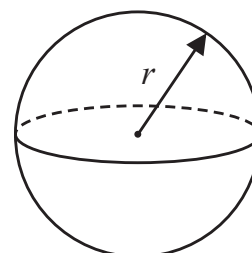= area of cross section × length

Volume of cylinder $= \pi r^2 h$
**Curved surface area
of cylinder** $= 2\pi rh$

Volume of sphere $= \dfrac{4}{3}\pi r^3$

Surface area of sphere $= 4\pi r^2$

Answer ALL TWENTY THREE questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

1 Brendon, Asha and Julie share some money in the ratios 3 : 2 : 6
   The **total** amount of money that Asha and Julie receive is $36

   Work out the amount of money that Brendon receives.

Asha and Julie : 2 + 6 = 8

36 ÷ 8 = 4·5 ① (1 part = 4·5)

Brendon : 3 × 4·5 = $ 13·50
                    ①

$ ___13·50___ ①

(Total for Question 1 is 3 marks)

2 Show that $3\frac{1}{5} \times 2\frac{5}{8} = 8\frac{2}{5}$

Convert to improper functions :

① $\frac{16}{5} \times \frac{21}{8} = \frac{336}{40}$ ①

$= 8\frac{16 \div 8}{40 \div 8}$

$= 8\frac{2}{5}$ ①

$\begin{array}{r} 8 \\ 40\overline{)336} \\ 320 \\ \hline 16 \end{array}$

(Total for Question 2 is 3 marks)

3

3 (a) Make a the subject of $d = g + 2ac$

$$d = g + 2ac$$
$$d - g = 2ac \quad \text{①}$$
$$2a = \frac{d - g}{c}$$
$$a = \frac{d - g}{2c} \quad \text{①}$$

$$a = \frac{d - g}{2c}$$

....

(2)

(b) Factorise fully $9ef - 12f$

$$9ef - 12f$$
$$f(9e - 12)$$
$$3f(3e - 4)$$

$$3f(3e - 4) \quad \text{②}$$

....

(2)

(c) Expand and simplify $(x + 2)(x - 5)$

$$(x + 2)(x - 5)$$

$$= x^2 - 5x + 2x - 10 \quad \text{①}$$

$$= x^2 - 3x - 10$$

$$x^2 - 3x - 10 \quad \text{①}$$

....

(2)

(d) Simplify fully $\dfrac{n^4 \times n^7}{n^5}$

$$\frac{n^4 \times n^7}{n^5} = n^{4 + 7 - 5} \quad \text{①} = n^6$$

$$n^6 \quad \text{①}$$

....

(2)

(Total for Question 3 is 8 marks)

4 $B = \{b, l, u, e\}$

$G = \{g, r, e, y\}$

$W = \{w, h, i, t, e\}$

(a) List all the members of the set

    (i) $B \cup G$

$$B \cup G = \{ b, l, u, e, g, r, y \}$$
↑ combine the two sets together

                                                      ①

                                          $\{ b, l, u, e, g, r, y \}$

    (ii) $W \cap G'$

$$G' = \{ b, l, u, w, h, i, t \}$$
$$W = \{ w, h, i, t, e \}$$
$$W \cap G' = \{ w, h, i, t \}$$

                                          ①

                                   $\{ w, h, i, t \}$

                                          (2)

Serena writes down the statement $B \cap G \cap W = \varnothing$

(b) Is Serena's statement correct?
    You must give a reason for your answer.

No, Serena is wrong because the letter e appears in all three sets. ①

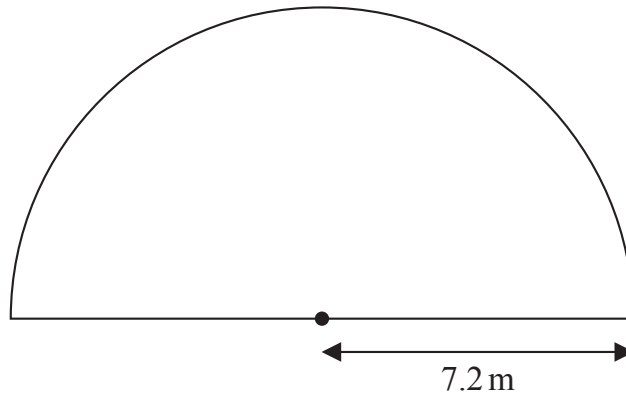   **(1)**

                                   (Total for Question 4 is 3 marks)

5 The diagram shows Yuen's garden.

Diagram **NOT** accurately drawn

7.2 m

The garden is in the shape of a semicircle of radius 7.2 m.
Yuen is going to cover his garden with grass seed.

Yuen has 12 boxes of grass seed.
Each box of grass seed contains enough seed to cover 6 m² of the garden.

Has Yuen enough grass seed for his garden?
Show your working clearly.

Area of semicircle = $\dfrac{\pi r^2}{2}$

Area of semicircle = $\dfrac{\pi (7.2)^2}{2}$ = 81.43 m² ①

Amount of grass seed = 12 × 6 = 72 m² ①

No, Yuen does not have enough grass seed for his garden. He only has enough grass seed to cover 72 m² which is less than 81.43 m². ①

6 Solve $x^2 - 5x - 36 = 0$
Show clear algebraic working.

$$x^2 - 5x - 36 = 0$$

$$①\ (x-9)(x+4) = 0$$

$$x - 9 = 0 \qquad \text{or} \qquad x + 4 = 0$$
$$x = 9 \qquad\qquad\qquad x = -4 \quad ①$$

$$-4, 9 \quad ①$$

(Total for Question 6 is 3 marks)

7 In a sale, the normal price of a hat is reduced by 15%
The sale price of the hat is 20.40 euros.

Work out the normal price of the hat.

$$100\% - 15\% = 85\%$$

$$\text{Convert to decimal} = \frac{85\%}{100} = 0.85$$

$$\text{Sale price} = \text{Initial price} \times 0.85$$

$$20.40 = \text{Initial price} \times 0.85$$

$$\text{Initial price} = \frac{20.40}{0.85} \quad ②$$

$$= 24 \text{ euros}$$

24 ① euros

(Total for Question 7 is 3 marks)

7

8 5 children are playing on a trampoline.
The mean weight of the 5 children is 28 kg.

2 of the children get off the trampoline.
The mean weight of these 2 children is 26.5 kg.

Work out the mean weight of the 3 children who remain on the trampoline.

Total weight of 5 children = 5 × 28 = 140 kg ①

Total weight of 2 children = 2 × 26.5 = 53 kg

Total weight of 3 children = 140 − 53 = 87 kg

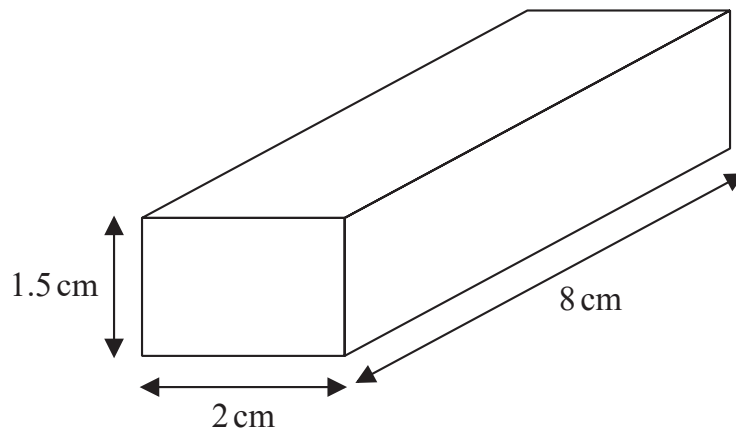Mean weight of 3 children = $\frac{87}{3}$ ①

= 29 kg ①

29 kg

(Total for Question 8 is 3 marks)

8

9   Pablo made a solid gold statue.

He melted down some gold blocks and used the gold to make the statue.
Each block of gold was a cuboid, as shown below.

Diagram **NOT**
accurately drawn

1.5 cm

8 cm

2 cm

The mass of the statue is 5.73 kg.
The density of gold is 19.32 g/cm^3

Work out the least number of gold blocks Pablo melted down in order to make the statue.
Show your working clearly.

Volume of gold blocks : 2 x 1.5 x 8 = 24 cm^3 ①

Mass of gold = density x volume

= 19.32 x 24

= 463.68 g ②

Convert to kg = 0.46368 kg

Number of gold blocks needed = $\dfrac{5.73}{0.46368}$

= 12.35 ... ①

round up the value

≈ 13

13 ①

(Total for Question 9 is 5 marks)

10 The diagram shows a regular hexagon, *ABCDEF*, and an isosceles triangle, *GHJ*.

Diagram **NOT**
accurately drawn

E *D* *J*

$(x + 5)$ cm

F *C*

A $(x - 1)$ cm *B* *G* $(2x - 3)$ cm *H*
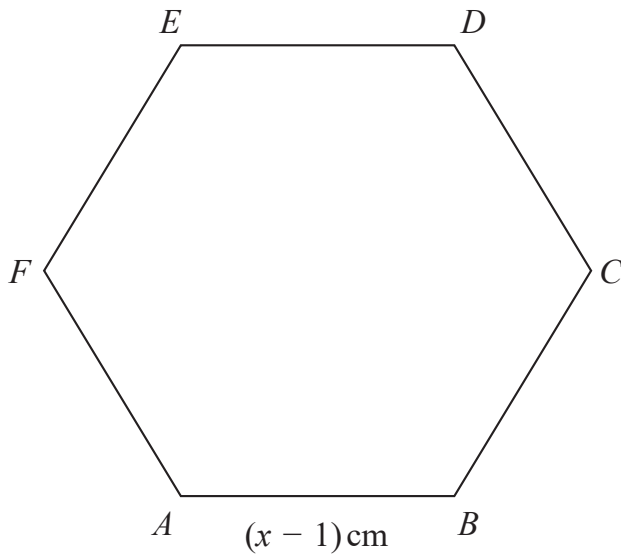
The perimeter of the hexagon is equal to the perimeter of the triangle.

Find the length of each side of the hexagon.
Show clear algebraic working.

Perimeter of hexagon = Perimeter of triangle

$$6(x-1) = (2x-3) + 2(x+5)$$

① $$6x - 6 = 2x - 3 + 2x + 10$$

① $$6x - 6 = 4x + 7$$ ①

$$6x - 4x = 7 + 6$$ ①

$$2x = 13$$

$$x = \frac{13}{2}$$

$$= 6.5$$

Side of hexagon $= 6.5 - 1$

$$= 5.5 \text{ cm}$$ ①

5·5 .... cm

(Total for Question 10 is 5 marks)

10

11 The weight of a cat is 4.3 kg correct to 2 significant figures.

(a) Write down the upper bound of the weight of the cat.

4.35 Ⓘ kg

(1)

(b) Write down the lower bound of the weight of the cat.

4.25 Ⓘ kg

(1)

$G = e - f$

$e = 17$ correct to the nearest integer
$f = 9.4$ correct to one decimal place
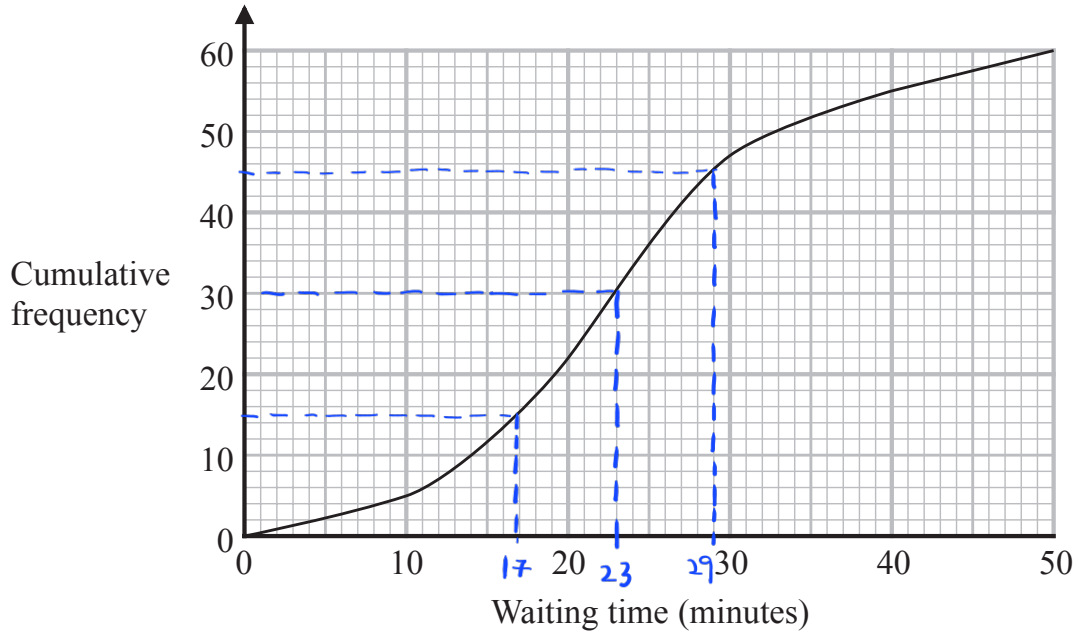
(c) Work out the upper bound for the value of G.

Upper bound of G = upper bound e − lower bound f

Upper bound of G = 17·5 − 9·35 Ⓘ

= 8·15

8·15 Ⓘ

(2)

(Total for Question 11 is 4 marks)

12 The cumulative frequency graph gives information about the waiting times, in minutes, of people with appointments at Hospital A.

(a) Use the graph to find an estimate of the median waiting time at Hospital A.

half of
cumulative frequency ↙

23 ① .... minutes

(1)

(b) Use the graph to find an estimate of the interquartile range of the waiting times at Hospital A.

Interquartile range = $Q_3 - Q_1$

IQR = 29 - 17 = 12 ①

12 ① .... minutes

(2)

At a different hospital, Hospital B, the median waiting time is 28 minutes and the interquartile range of the waiting times is 19 minutes.

(c) Compare the waiting times at Hospital A with the waiting times at Hospital B.

Hospital A has a lower waiting time than Hospital B because its
① median is lower than Hospital B. The waiting time for Hospital A is
less spread than Hospital B because the interquartile range is lower.
①

(2)

(Total for Question 12 is 5 marks)

13 (a) Use algebra to show that $0.5\dot{7}\dot{2} = \dfrac{63}{110}$

Let x be $= 0.57272\ldots$

$100x = 57.272\ldots$ ①

$100x - x = 57.272\ldots - 0.572\ldots$

$99x = 56.7$

$x = \dfrac{56.7}{99}$ ⓑ

$x = \dfrac{63}{110}$ (shown)

(2)

Given that y is a prime number,

(b) express $\dfrac{3}{2 - \sqrt{y}}$ in the form $\dfrac{a + b\sqrt{y}}{c - y}$ where a, b and c are integers.
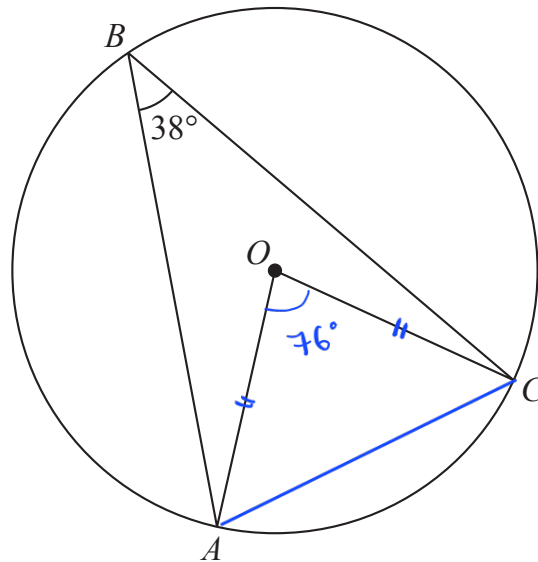
$\dfrac{3}{2 - \sqrt{y}} \times \dfrac{(2 + \sqrt{y})}{(2 + \sqrt{y})}$ ① $= \dfrac{3(2 + \sqrt{y})}{(2 - \sqrt{y})(2 + \sqrt{y})}$

$= \dfrac{6 + 3\sqrt{y}}{4 - y}$

$\dfrac{6 + 3\sqrt{y}}{4 - y}$ ①

(2)

(Total for Question 13 is 4 marks)

13

14

Diagram **NOT** accurately drawn

A, *B* and *C* are points on a circle, centre *O*.
Angle *ABC* = 38°

Work out the size of angle *OAC*.
Give a reason for each stage of your working.

$\angle AOC = 38 \times 2 = 76°$ ①

(Angle at centre is twice the angle at circumference) ①

$\angle OAC = \dfrac{180° - 76°}{2}$

$= 52°$

(Angle in a triangle adds up to 180°, base of isosceles are equal) ①

52 ① °

(Total for Question 14 is 4 marks)

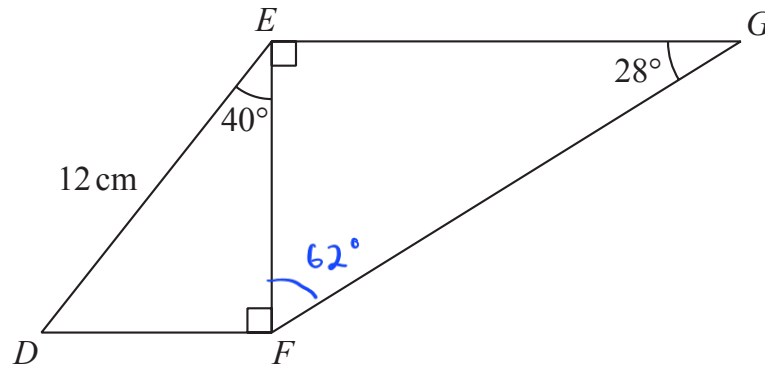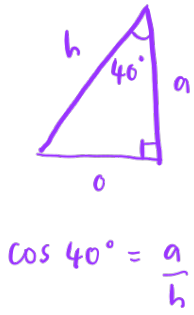**15** The diagram shows two right-angled triangles, *DEF* and *EFG*.

Diagram **NOT** accurately drawn

(student annotation: triangle with angle 40°, sides h, a, o)

$$\cos 40° = \frac{a}{h}$$

(labels on main diagram: E, G, 28°, 40°, 12 cm, 62°, D, F)

Work out the length of *EG*.
Give your answer correct to 3 significant figures.

$$EF = 12 \cos 40° \quad ①$$
$$= 9.1925 \text{ cm} \quad ①$$

$$\angle GFE = 180° - 90° - 28°$$
$$= 62°$$

$$\tan 62° = \frac{EG}{9.1925} \quad ①$$

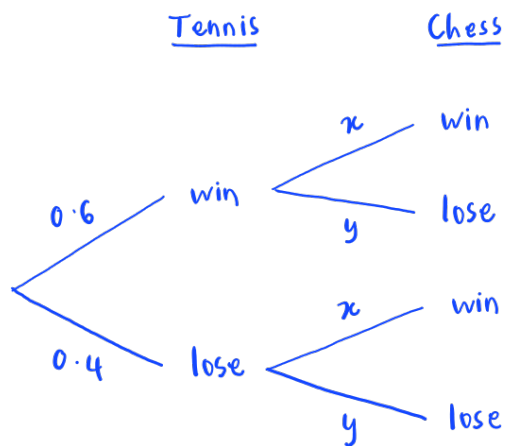$$EG = 9.1925 \times \tan 62°$$
$$= 17.3 \ (3sf)$$

17.3 ① cm

(Total for Question 15 is 4 marks)

16 Steffi is going to play one game of tennis and one game of chess.

The probability that she will win the game of tennis is 0.6
The probability that she will win **both** games is 0.42

Work out the probability that she will **not** win either game.

Tennis   Chess

$0.6 \times x = 0.42$

$x = 0.7$ ①

$y = 1 - 0.7$

$= 0.3$ ①

$P(\text{lose}, \text{lose}) = 0.4 \times 0.3$ ①

$= 0.12$ ①

0.12

(Total for Question 16 is 4 marks)

16

17 The function f is such that $f(x) = (x - 4)^2$ for all values of x.

(a) Find f(1)

$$f(x) = (x-4)^2$$
$$f(1) = (1-4)^2$$
$$= (-3)^2$$
$$= 9$$

9 ①

(1)

(b) State the range of the function f.

$f(x) \geq 0$ ①

(1)

The function g is such that $g(x) = \dfrac{4}{x+3}$ $x \neq -3$

(c) Work out fg(2)

$$f(x) = (x-4)^2$$
$$g(x) = \frac{4}{x+3}$$
$$fg(x) = \left(\frac{4}{x+3} - 4\right)^2$$
$$fg(2) = \left(\frac{4}{2+3} - 4\right)^2 \text{ ①}$$
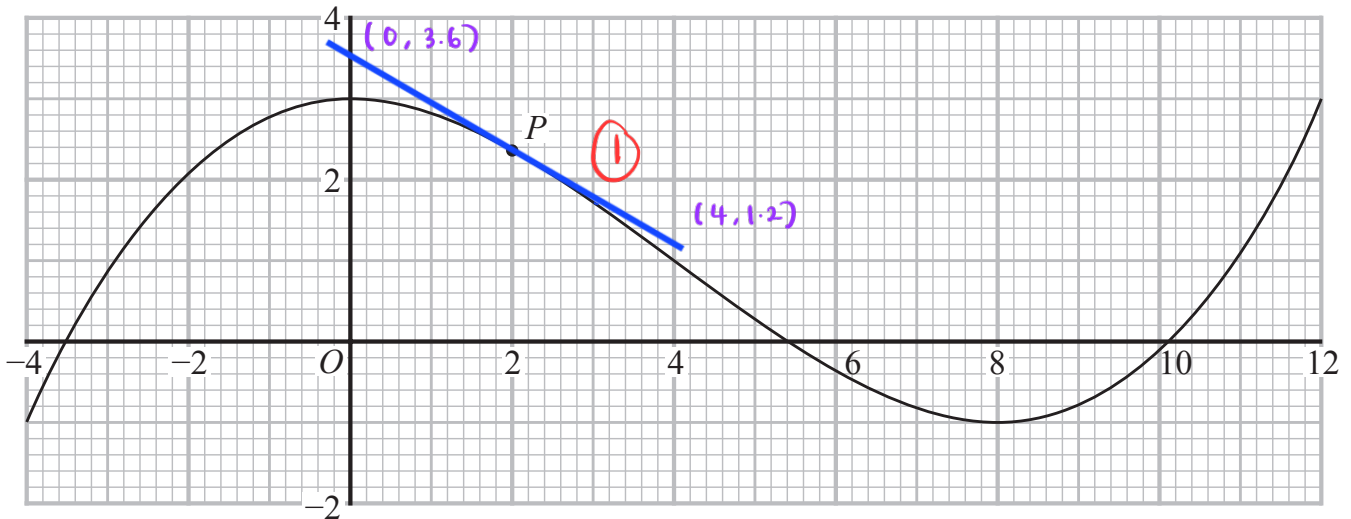$$= \left(-\frac{16}{5}\right)^2 = \frac{256}{25}$$

$\dfrac{256}{25}$ ①

(2)

(Total for Question 17 is 4 marks)

18 The diagram shows the graph of $y = f(x)$ for $-4 \leqslant x \leqslant 12$

The point P on the curve has x coordinate 2

(a) (i) Use the graph to find an estimate for the gradient of the curve at P.

$$m = \frac{3.6 - 1.2}{0 - 4} \; ①$$

$$= -0.6$$

.... -0.6 ①

(3)

(ii) Hence find an equation of the tangent to the curve at P.
Give your answer in the form $y = mx + c$

$$y = mx + c \quad \leftarrow y\text{-intercept}$$
$$y = -0.6x + 3.6 \quad ①$$

y-intercept $= 3.6$

.... ①
$y = -0.6x + 3.6$

(2)

The equation $f(x) = k$ has exactly two different solutions for $-4 \leqslant x \leqslant 12$

(b) Use the graph to find the two possible values of k.

.... -1 ① , ... 3 ①

(2)

(Total for Question 18 is 7 marks)

18

19
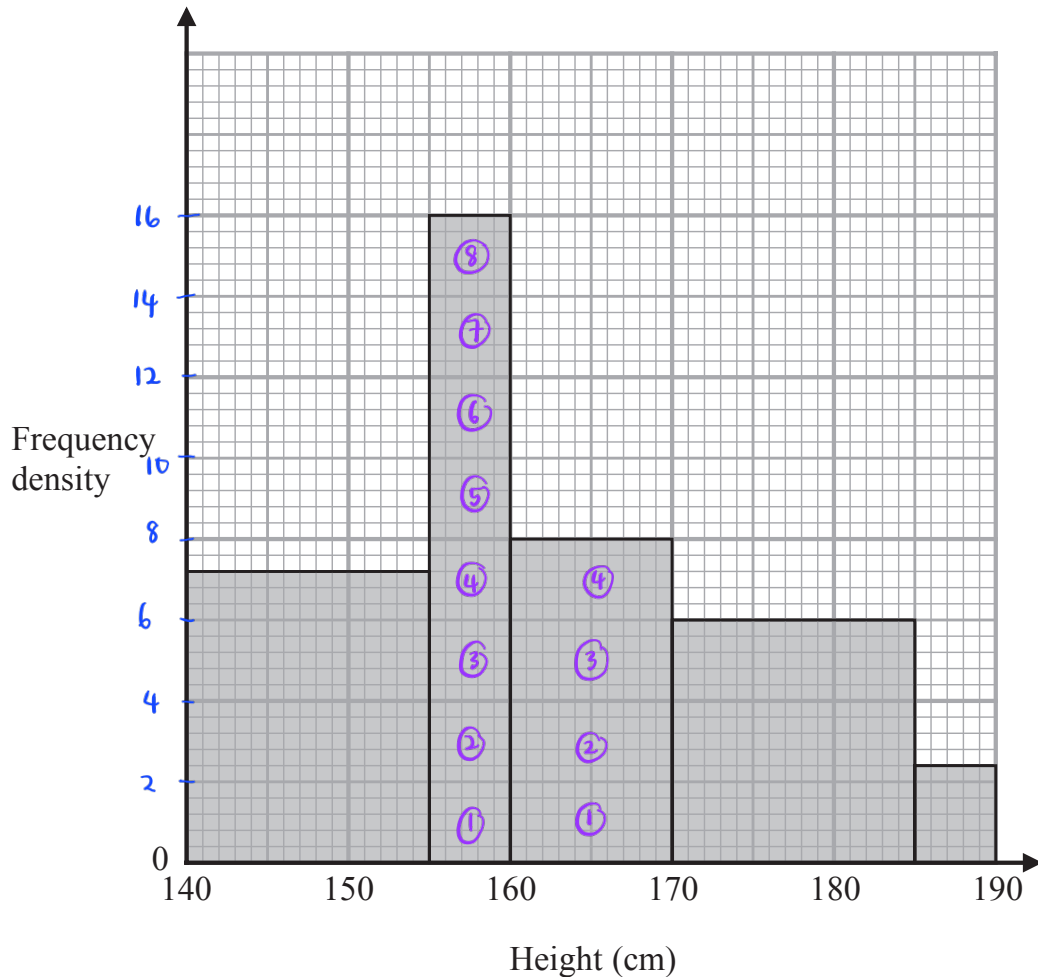
Height (cm)

The histogram gives information about the heights of all the Year 11 students at a school.

There are 160 students in Year 11 with a height between 155 cm and 170 cm.

Work out the total number of students in Year 11 at the school.

Frequency density = $\dfrac{\text{Frequency}}{\text{Class width}}$

Frequency = Class width × frequency density

$160 = 5 \times 8x + 10 \times 4x$ ①

$160 = 40x + 40x$

$160 = 80x$

$x = 2$ ①

Total Students : $15 \times 7.2 + 5 \times 16 + 10 \times 8 + 15 \times 6 + 5 \times 2.4$ ①

$= 370$

370 ①

(Total for Question 19 is 4 marks)

20 The diagram shows a frustum of a cone and a sphere.

The frustum is made by removing a small cone from a large cone.
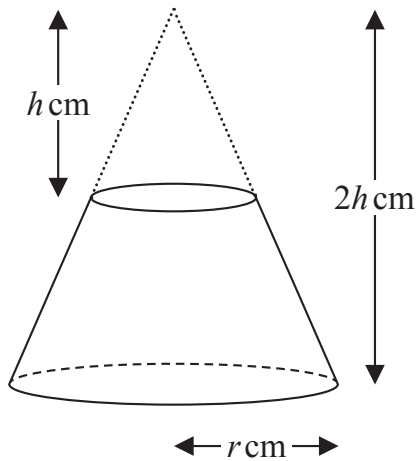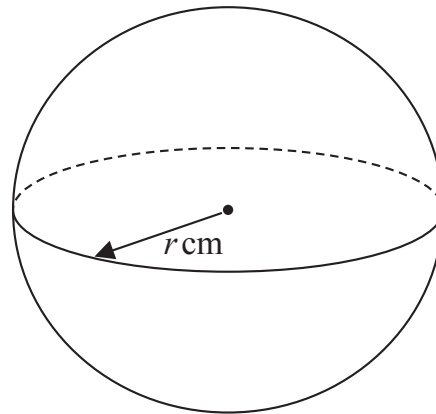The cones are similar.

Diagram **NOT** accurately drawn

The height of the small cone is h cm.
The height of the large cone is $2h$ cm.
The radius of the base of the large cone is r cm.

The radius of the sphere is r cm.

Given that the volume of the frustum is equal to the volume of the sphere,

find an expression for r in terms of h.
Give your expression in its simplest form.

Volume of cone : $\dfrac{1}{3}\pi r^2 h$

Volume of sphere $= \dfrac{4}{3}\pi r^3$

Volume of big cone $= \dfrac{1}{3}\pi r^2 (2h)$

$= \dfrac{2\pi r^2 h}{3}$ ①

Volume of small cone $= \dfrac{1}{3}\pi \left(\dfrac{1}{2}r\right)^2 (h)$

$= \dfrac{1}{3}\pi \left(\dfrac{1}{4}r^2\right) h$

$= \dfrac{1}{12}\pi r^2 h$

Volume of frustum = Volume of big cone - volume of small cone

Volume of frustum $= \dfrac{2\pi r^2 h}{3} - \dfrac{1}{12}\pi r^2 h$

$= \dfrac{7}{12}\pi r^2 h$ ①

Volume of sphere $= \dfrac{4}{3}\pi r^3$

① $\dfrac{4}{3}\pi r^3 = \dfrac{7}{12}\pi r^2 h$ ①

$\dfrac{4}{3}r = \dfrac{7}{12}h$

$4 \times 12\, r = 7 \times 3\, h$

$48\, r = 21h$

$r = \dfrac{21h}{48}$
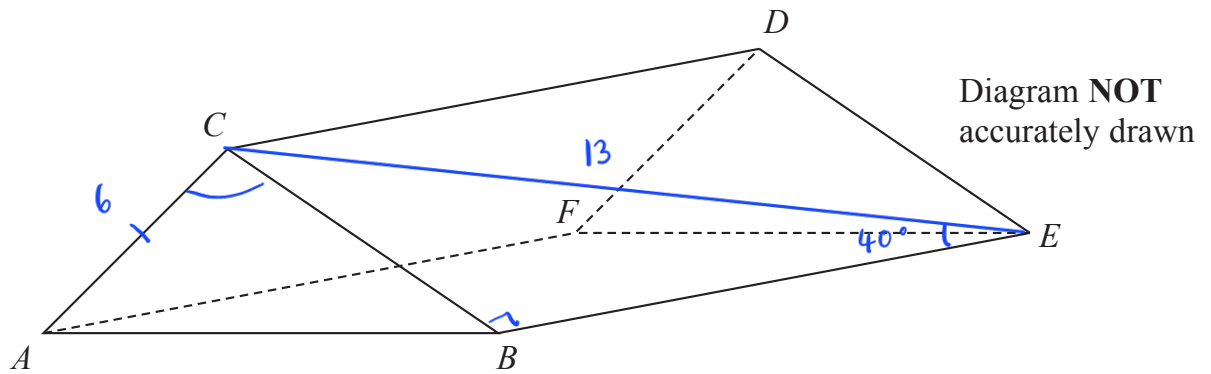
$r = \dfrac{7}{16}h$

$r = \underline{\dfrac{7}{16}h}$ ①

(Total for Question 20 is 5 marks)

21 The diagram shows the prism *ABCDEF* with cross section triangle *ABC*.

Diagram **NOT** accurately drawn

Angle *BEC* = 40° and angle *ACB* is obtuse.
AC = 6 cm and *CE* = 13 cm

The area of triangle *ABC* is 22 cm²

Calculate the length of *AB*.
Give your answer correct to one decimal place.

$$CB = 13 \sin 40°$$
$$= 8.3562 \dots \quad ①$$

Area of triangle $= \frac{1}{2} ab \sin C$

$$22 = \frac{1}{2} \times 6 \times 8.3562\dots \sin \angle ACB \quad ①$$

$$\sin \angle ACB = 0.87758 \dots$$

Acute version $ACB = \sin^{-1}(0.87758\dots)$

$$= 61.353° \quad ①$$

$$ACB = 180 - 61.353 = 118.647° \quad ①$$

cosine rule $\rightarrow a^2 = b^2 + c^2 - 2bc \cos A$

$$(AB)^2 = 6^2 + (8.3562)^2 - 2(6)(8.3562) \cos 118.647°$$

$$= 153.899 \quad ①$$

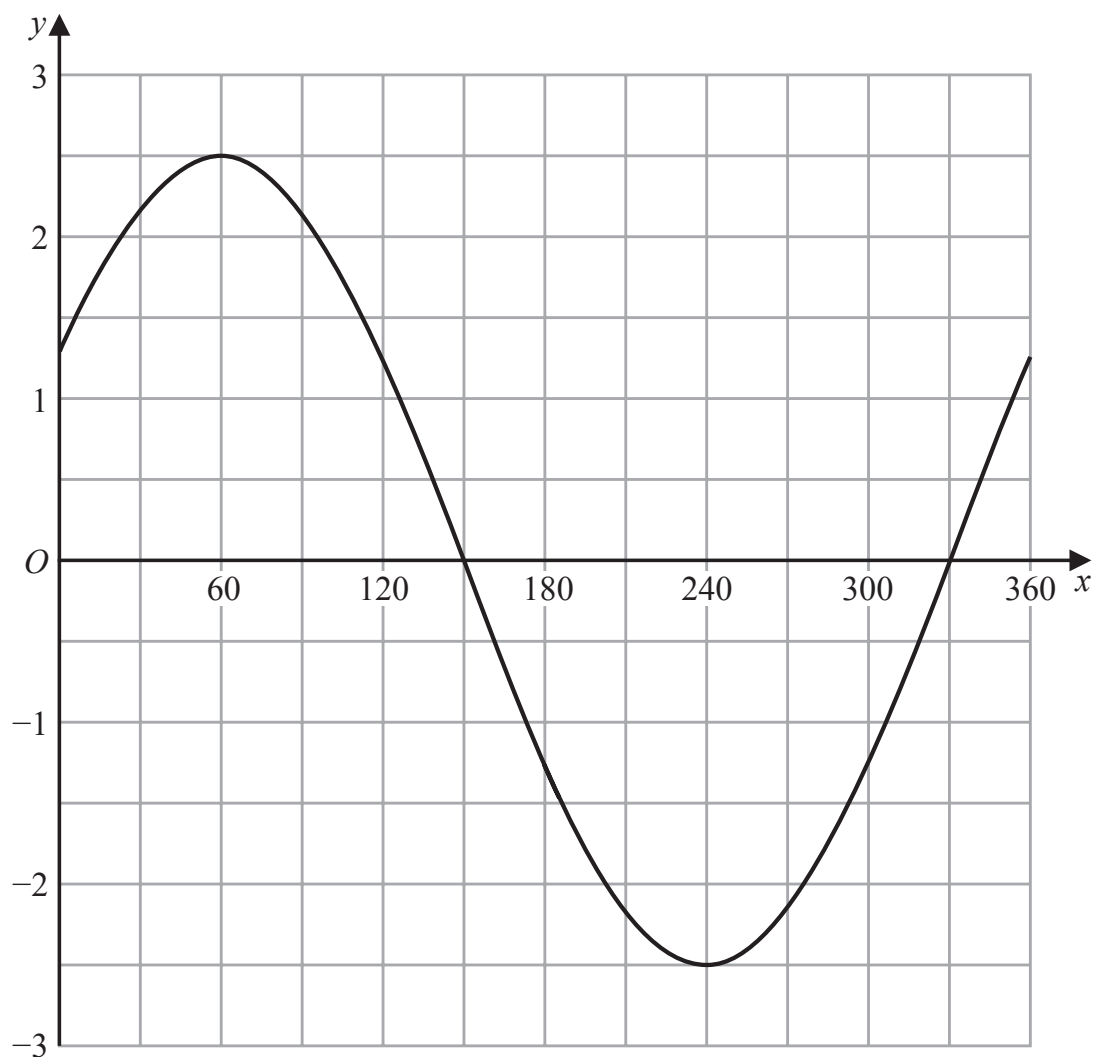$$AB = \sqrt{153.899}$$

$$= 12.4 \text{ cm} \quad ①$$

.... 12.4 .... cm

(Total for Question 21 is 6 marks)

22 The graph of $y = a\cos(x + b)^\circ$ for $0 \leqslant x \leqslant 360$ is drawn on the grid.

(a) Find the value of a and the value of b.

If $b > 0$, then curve shifts to the left

If $b < 0$, then curve shifts to the right

$a = \underline{\quad 2.5 \quad}$ ①

$b = \underline{\quad -60 \quad}$ ①

(2)

Another curve C has equation $y = f(x)$
The coordinates of the minimum point of C are $(4, 5)$

(b) Write down the coordinates of the minimum point of the curve with equation

(i) $y = f(2x)$

$\dfrac{4}{2} = 2$ y-coordinate
             unaffected

$(\underline{\quad 2 \quad}, \underline{\quad 5 \quad})$ ①

(ii) $y = f(x) - 7$

$5 - 7 = -2$ x-coordinate
             unaffected

$(\underline{\quad 4 \quad}, \underline{\quad -2 \quad})$ ①

(2)

(Total for Question 22 is 4 marks)

23 A particle moves along a straight line.
The fixed point O lies on this line.
The displacement of the particle from O at time t seconds, $t \geqslant 0$, is s metres where

$$s = t^3 + 4t^2 - 5t + 7$$

At time T seconds the velocity of P is V m/s where $V \geqslant -5$

Find an expression for T in terms of V.

Give your expression in the form $\dfrac{-4 + \sqrt{k + mV}}{3}$ where k and m are integers to be found.

$$v = \frac{ds}{dt}$$

$$v = 3t^2 + 8t - 5 \quad ①$$

$$v = 3T^2 + 8T - 5$$

$$0 = 3T^2 + 8T - 5 - v \quad ①$$

$$= \frac{-8 \pm \sqrt{(8)^2 - 4(3)(-5-v)}}{2(3)} \quad ① \qquad \qquad \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-8 \pm \sqrt{64 + 60 + 12v}}{6}$$

$$= \frac{-8 \pm \sqrt{124 + 12v}}{6} \quad ①$$

$$= \frac{-8 \pm \sqrt{4(31 + 3v)}}{6}$$

$$= \frac{-8 \pm 2\sqrt{(31 + 3v)}}{6} \quad ① \qquad \qquad \therefore \ \frac{-4 + \sqrt{31 + 3V}}{3}$$

$$= \frac{-4 \pm \sqrt{31 + 3v}}{3} \qquad \qquad \qquad T = \frac{-4 + \sqrt{31 + 3v}}{3} \quad ①$$

(Total for Question 23 is 6 marks)

TOTAL FOR PAPER IS 100 MARKS